CALCULATION OF SHOCK WAVES IN A FLOW OF MOIST VAPOR AT LOW PRESSURE

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A method is given for calculating oblique shocks in a region of moist vapor, when conditions of phase equilibrium are satisfied in the shock process.

When superheated vapor arrives in a nozzle, and a shock wave follows a condensation shock, the moisture moving in the stream is finely dispersed and has a velocity close to that of the vapor phase. It is natural to suppose that in the presence of a large number of vapor-forming centers, the shock wave occurs in conditions of phase equilibrium. This proposition is valid right up to complete evaporation of the moisture. It should be noted that such shocks are met comparatively frequently. In the shock analysis the following assumptions are made.

The Clapeyron equation $pV = RT$ is applicable to the vapor phase.

The velocity of the moisture drops generated in the condensation shock is equal to that of the vapor. This proposition is valid both ahead of the shock wave and behind it.

In the low pressure region it is assumed that the saturation temperature and the value of the latent heat of vaporization do not change in passing through the shock. For water vapor this condition may be assumed for static pressures behind the shock of less than $0.98 \cdot 10^5$ N/m^2 ($p_1 \leq 0.98 \cdot 10^5$ N/m^2).

Taking these assumptions into account, the basic equations of gasdynamics may be written in the following form:

equation of continuity

$$\frac{M_1 \sin\beta_1}{M_2 \sin\beta_2} = \frac{x_1}{x_2}\,\frac{T_1}{T_2}\,\frac{p_2}{p_1}, \qquad (1)$$

where $M_1 = c_1/\sqrt{kRT_1}$ and $M_2 = c_2/\sqrt{kRT_1}$ are dimensionless flow velocities;

momentum equation components normal to the shock front

$$\frac{p_2}{p_1} - 1 = \frac{k}{x_1} M_1 \sin\beta_1 (M_1 \sin\beta_1 - M_2 \sin\beta_2); \qquad (2)$$

momentum equation along the shock front

$$M_1 \cos\beta_1 = M_2 \cos\beta_2; \qquad (3)$$

energy equation, taking into account that $r_1 = r_2$

$$i_0 - \frac{kRT_1}{2} M_1^2 \cos\beta_1 = rx_1 + \frac{kRT_1}{2} M_1^2 \sin^2\beta_1 =$$

$$= 2x_2 + \frac{kRT_1}{2} M_2^2 \sin^2\beta_2.$$

The basic equation for calculating an adiabatic shock is found as follows.

From the first two equations we may obtain ($T_1 = T_2$)

$$M_1 \sin\beta_1 \frac{x_2}{x_1}\,\frac{p_1}{p_2} = M_1 \sin\beta_1 - \frac{x_1}{k M_1 \sin\beta_1}\left(\frac{p_2}{p_1} - 1\right). \qquad (4)$$

The energy equation gives

$$x_2 = x_1 + \frac{1}{2}\,\frac{kRT_1}{r}(M_1^2 \sin^2\beta_1 - M_2^2 \sin^2\beta_2),$$

or, taking into account momentum equation (2),

$$x_2 = x_1 + \frac{1}{2}\,\frac{RT_1}{r}\cdot\frac{x_1}{M_1 \sin\beta_1}\left(\frac{p_2}{p_1} - 1\right) \times$$

$$\times \left[2M_1 \sin\beta_1 - \frac{x_1}{k M_1 \sin\beta_1}\left(\frac{p_2}{p_1} - 1\right)\right].$$

Thus, Eq. (4) is brought to the form

$$M_1 \sin\beta_1 + \frac{1}{2}\,\frac{RT_1}{r}\left(\frac{p_2}{p_1} - 1\right)\cdot$$

$$\cdot\left[2M_1 \sin\beta_1 - \frac{x_1}{k M_1 \sin\beta_1}\left(\frac{p_2}{p_1} - 1\right)\right] =$$

$$= \frac{p_2}{p_1}\left[M_1 \sin\beta_1 - \frac{x_1}{k M_1 \sin\beta_1}\left(\frac{p_2}{p_1} - 1\right)\right],$$

or

$$\frac{p_2}{p_1} = \left(1 - \frac{RT_1}{r}\right) - \frac{x_1}{k M_1^2 \sin^2\beta_1}\,\frac{1}{2}\,\frac{RT_1}{r} \times,$$

$$\times \left[\frac{x_1}{k M_1^2 \sin^2\beta_1}\left(1 - \frac{1}{2}\,\frac{RT_1}{r}\right)\right]^{-1}. \qquad (5)$$

From the last equation it is easy to calculate the value of p_2/p_1 for the given flow parameters ahead of the shock p_1, T_1 and M_1, β_1. For convenience of calculation, Fig. 1 shows a diagram for calculating shock waves in moist water vapor. Furthermore, from the momentum equation we have

$$M_2 \sin\beta_2 = M_1 \sin\beta_1 - \frac{x_1}{k M_1 \sin\beta_1}\left(\frac{p_2}{p_1} - 1\right).$$

The degree of dryness of the vapor behind the shock may be determined from the continuity equation

$$x_2 = \frac{M_2 \sin\beta_2}{M_1 \sin\beta_1}\,\frac{p_2}{p_1}\,x_1.$$

The deflection angle of the flow δ is determined from (2) and (3). On the basis of these equations we obtain

$$\operatorname{tg}\beta_2=\operatorname{tg}\beta_1-\frac{x_1}{k\,M_1^2\sin^2\beta_1\cos\beta_1}\left(\frac{p_2}{p_1}-1\right),$$

or, after simple transformations,

$$\operatorname{tg}\delta=\frac{1}{\operatorname{tg}\beta_1}\left\{\frac{x_1}{k\,M_1^2}\left(\frac{p_2}{p_1}-1\right)\left[1-\frac{x_1}{kM_1^2}\left(\frac{p_2}{p_1}-1\right)\right]^{-1}\right\}. \quad (6)$$

Taking (5) into account, we obtain

$$\operatorname{tg}\delta=\left(1-\frac{RT_1}{r}-\frac{x_1}{k\,M_1^2\sin^2\beta_1}\right)\left(\frac{1}{\sin^2\beta_1}\left[1-\frac{1}{2}\times\right.\right.$$

$$\left.\times\frac{RT_1}{r}\right]\left\{1-\left(1-\frac{RT_1}{r}-\frac{x_1}{k\,M_1^2\sin^2\beta_1}\right)\times\right.$$

$$\left.\left.\times\left[\frac{1}{\sin^2\beta_1}\left(1-\frac{1}{2}\frac{RT_1}{r}\right)\right]^{-1}\right\}\operatorname{tg}\beta_1\right)^{-1},$$

or

$$\operatorname{tg}\delta=\left(1-\frac{RT_1}{r}-\frac{x_1}{k\,M_1^2\sin^2\beta_1}\right)\left\{\left[\frac{1}{\sin^2\beta_1}\left(1-\frac{1}{2}\times\right.\right.\right.$$

$$\left.\left.\left.\times\frac{RT_1}{r_1}\right)-\left(1-\frac{RT_1}{r}\right)+\frac{x_1}{k\,M_1^2\sin^2\beta_1}\right]\operatorname{tg}\beta_1\right\}^{-1}. \quad (6')$$

When values of M_1, p_1, x_1 and the deflection angle δ are the given quantities, the problem is complicated somewhat. For these conditions we may find the angle of inclination β_1 of the shock from (6'), but this requires much calculation. The following method is more efficient.

Assuming a number of values of $\beta_1 > \arcsin(1/M_1)$ the ratio p_2/p_1 is determined from quantity $x_1/kM_1^2\sin^2\beta_1$ and the static pressure p_1 using the shock diagram. From (6') we determine the flow deflection angle δ, and draw the graphical relation $\delta = f(\beta_1)$.

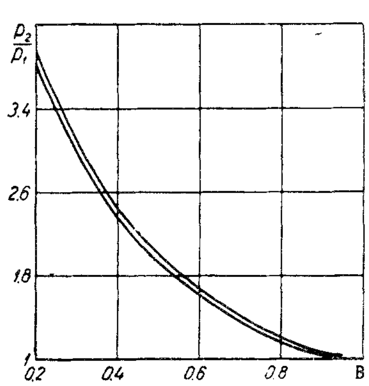

Fig. 1. Dependence of p_2/p_1 on the group $B \equiv x_1/kM_1^2\sin^2\beta_1$.

The intersection of this curve with the given value of δ gives the desired shock inclination angle. The subsequent calculation does not present difficulties and is carried out in the sequence indicated above.

We note that for each flow deflection angle there are two solutions for angle β_1 and pressure ratio p_2/p_1, as is well known in the gasdynamics of a single-phase fluid. The regions of application of the strong and weak solutions for an inclined shock in a two-phase medium are evidently similar to the regions of application of these solutions in a single-phase fluid.

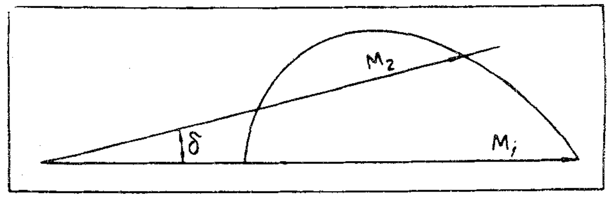

Fig. 2. Shock polar for moist water vapor.

To illustrate these conclusions Fig. 2 shows the shock polar in moist water vapor with $p_1 = 0.98\cdot10^5$ N/m^2, $x_1 = 0.90$ and $M_1 = 1.5$. We note that the region of application of the method examined has a limit in x_2, the limit value being 1. Above this limit there is no single-valued relation between pressure and temperature behind the shock. We shall establish this limit.

With $x_2 = 1$, equations (1) and (2) give

$$\frac{p_1}{p_2}\,\frac{M_1\sin\beta_1}{x_1}=M_1\sin\beta_1-\frac{x_1}{k\,M_1\sin\beta_1}\left(\frac{p_2}{p_1}-1\right), \quad (7)$$

whence we obtain

$$M_1\sin\beta_1=\sqrt{\frac{x_1}{k}\left(\frac{p_2}{p_1}-1\right)\Big/\left(1-\frac{1}{x_1}\,\frac{p_1}{p_2}\right)}. \quad (7')$$

The energy equation with $x_2 = 1$ gives

$$r(1-x_1)=\frac{RT_1}{2}\,\frac{x_1}{M_1\sin\beta_1}\left(\frac{p_2}{p_1}-1\right)\times$$

$$\times\left[2M_1\sin\beta_1-\frac{x_1}{k\,M_1\sin\beta_1}\left(\frac{p_2}{p_1}-1\right)\right].$$

Taking (7) into account, we obtain

$$r(1-x_1)=\frac{RT_1}{2}\left(\frac{p_2}{p_1}-1\right)\left(x_1+\frac{p_1}{p_2}\right).$$

After simple transformations we have

$$(p_2/p_1)^2-2(r/RT_1-1/2)(1-x_1)\,p_2/p_1-1/x_1=0.$$

The solution of this equation will be

$$p_2/p_1=(r/RT_1-1/2)(1-x_1)+$$

$$+\sqrt{(r/RT_1-1/2)^2(1-x_1)^2+1/x_1}. \quad (8)$$

Thus the value of $M_1\sin\beta_1$ at which the shock process ends on the upper boundary curve, will be expressed as

$$M_1^2\sin^2\beta_1=\left\{\frac{x_1^2}{k}\left[\left(\frac{r}{RT_1}-\frac{1}{2}\right)(1-x_1)-1+\right.\right. \quad (9)$$

$$+\sqrt{\left(\frac{r}{RT_1}-\frac{1}{2}\right)^2(1-x_1)^2+\frac{1}{x_1}}\Bigg]\left[\left(\frac{r}{RT_1}-\frac{1}{2}\right)\times\right.$$

$$\times(1-x_1)+\sqrt{\left(\frac{r}{RT_1}-\frac{1}{2}\right)^2(1-x_1)^2+\frac{1}{x_1}}\Bigg]\Bigg\}\times$$

$$\times\left\{x_1\left[\left(\frac{r}{RT_1}-\frac{1}{2}\right)(1-x_1)+\right.\right.$$

$$\left.\left.+\sqrt{\left(\frac{r}{RT_1}-\frac{1}{2}\right)^2(1-x_1)^2+\frac{1}{x_1}}\right]-1\right\}^{-1}. \quad (9)\ \text{(cont'd)}$$

The results of the calculations are shown in Fig. 3.

From (6), for the given parameters p_1, x_1, and M_1, we can calculate the flow deflection angle at which

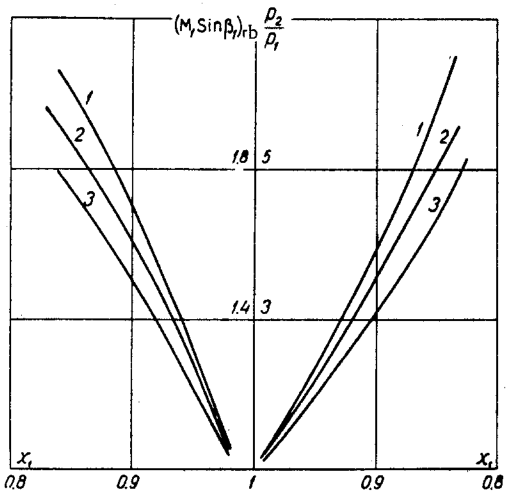

Fig. 3. Determination of boundary values of the quantity $M_1\sin\beta_1$: 1) for $p_1 = 10^3$ N/m^2; 2) 10^4; 3) $5 \cdot 10^4$.

the vapor is dry and saturated behind the shock. For this purpose, we determine the quantity $M_1 \sin\beta_1$ from the last relation, which allows us to determine the angle β_1, while the pressure ratio p_2/p_1 is determined from (8). It is evident that if $\delta < \delta_b$, the shock process ends in the region of moist vapor. Analogously, if $M_1 \sin\beta_1 < (M_1 \sin\beta_1)_b$, the process also proceeds without intersecting the upper boundary curve.

If $M_1 \sin\beta_1 > (M_1 \sin\beta_1)_b$, the vapor is superheated in the end state (behind the shock). There is then no single-valued relation between p_2 and T_2.

Assuming that for superheated and saturated vapor the value of the enthalpy is determined as [1]

$$\Delta i = c_p \Delta T$$

we obtain from the energy equation

$$T_2 = T_1 - (1-x_1)\frac{r_1}{c_p} + $$

$$+\frac{1}{2}\,\frac{kRT_1}{c_p}(M_1^2\sin^2\beta_1 - M_2^2\sin^2\beta_2). \quad (10)$$

For the case examined the equations of continuity and momentum give

$$\frac{M_1\sin\beta_1}{x_1}\,\frac{T_2}{T_1}\,\frac{p_1}{p_2} = M_1\sin\beta_1 - \frac{x_1}{kM_1\sin\beta_1}\left(\frac{p_2}{p_1}-1\right). \quad (11)$$

Substituting T_2 from (10) into this equation, and bearing in mind that, according to the momentum equation

$$M_1^2\sin^2\beta_1 - M_2^2\sin\beta_2 = \frac{x_1}{kM_1\sin\beta_1}\left(\frac{p_2}{p_1}-1\right)\times$$

$$\times\left[2M_1\sin\beta_1 - \frac{x_1}{kM_1\sin\beta_1}\left(\frac{p_2}{p_1}-1\right)\right],$$

we obtain

$$T_1-(1-x_1)\frac{r}{c_p}+\frac{R}{2c_p}\left[2M_1\sin\beta_1-\frac{x_1}{kM_1\sin\beta_1}\times\right.$$

$$\left.\times\left(\frac{p_2}{p_1}-1\right)\right]\times\frac{x_1}{kM_1\sin\beta_1}\left(\frac{p_2}{p_1}-1\right)=\frac{p_2}{p_1}\,\frac{x_1T_1}{M_1\sin\beta_1}\times$$

$$\times\left[M_1\sin\beta_1-\frac{x_1}{kM_1\sin\beta_1}\left(\frac{p_2}{p_1}-1\right)\right].$$

After simple transformations

$$\left(\frac{p_2}{p_1}-1\right)^2-\frac{c_v k M_1^2\sin^2\beta_1/x_1 - c_p}{c_p-R/2}\left(\frac{p_2}{p_1}-1\right)-$$

$$-\frac{1-x_1}{x_1^2}\,\frac{(r_1-c_pT_1)}{(c_p-R/2)T_1}kM_1^2\sin^2\beta_1=0.$$

The solution of this equation for p_2/p_1 will be

$$\frac{p_2}{p_1}=1+\frac{k}{k+1}\left(\frac{M_1^2\sin^2\beta_1}{x_1}-1\right)+$$

$$+\left[\left(\frac{k}{k+1}\right)^2\left(\frac{M_1^2\sin^2\beta_1}{x_1}-1\right)^2+\right.$$

$$\left.+2\,\frac{1-x_1}{x_1^2}\,\frac{k^2}{k+1}\left(\frac{r}{c_pT_1}-1\right)M_1^2\sin^2\beta_1\right]^{1/2}. \quad (12)$$

The minus sign in front of the radical corresponds to the unreal case of decrease of pressure in the shock, and so we shall not examine that solution. In the case when $x_1 = 1$, we can at once obtain the known relation for the usual adiabatic shock

$$\frac{p_2}{p_1}=\frac{2k}{k+1}(M_1^2\sin^2\beta_1-1)+1.$$

The subsequent calculation does not present any difficulty. From (11), (6), (2), (3) we may successively determine T_2, the flow deflection angle δ; $M_2\sin\beta_2$; $M_2\cos\beta_2$, and finally $\beta_2 = \beta_1-\delta$.

When the state of the flow behind the shock corresponds to superheated vapor, and the quantities assigned for the calculation are M_1, p_1, x_1 and the flow deflection angle δ, the shock is calculated in the following sequence.

From the formula

$$\operatorname{tg}\delta\operatorname{tg}\beta_1=\frac{x_1}{kM_1^2}\left[\frac{k}{k+1}\left(\frac{M_1^2\sin^2\beta_1}{x_1}-1\right)+\right.$$

$$+\left[\left(\frac{k}{k+1}\right)^2\left(\frac{M_1^2\sin^2\beta_1}{x_1}-1\right)^2+\right.$$

$$\left.\left.+\frac{(1-x_1)k^2}{x_1^2(k+1)}\left(\frac{r}{c_pT_1}-1\right)M_1^2\sin^2\beta_1\right]^{1/2}\right]\times$$

$$\times\left\{1-\frac{x_1}{kM_1^2}\left[\frac{k}{k+1}\left(\frac{M_1^2\sin^2\beta_1}{x_1}-1\right)+\right.\right.$$

$$+\left[\left(\frac{k}{k+1}\right)^2\left(\frac{M_1^2\sin^2\beta_1}{x_1}-1\right)^2+\right.$$

$$\left.\left.\left.+\frac{(1-x_1)k^2}{x_1^2(1+k)}\left(\frac{r}{c_pT_1}-1\right)M_1^2\sin^2\beta_1\right]^{1/2}\right]\right\}^{-1},$$

obtained on the basis of (6) and (11), we may determine the angle β_1. For this we construct the relation $\delta = f(\beta_1)$ with the given values M_1, p_1, x_1, and determine the angle β_1 from the point of intersection of this curve with the given value of δ. The subsequent calculation is carried out in the order indicated for the case when the calculation was done from given values of p_1, x_1, M_1 and β_1.

We note that for moist vapor, and also for gas, there is some limit value of the deflection angle, δ_{max}. In contrast with the gas case, the maximum deflection angle depends not only on the number M_1, but also on the static pressure p_1, and on the initial degree of dryness of the stream, x_1. This applies equally to shock processes ending in the moist vapor region, and to shock waves leaving the flow in the superheated vapor region. Thus, in the case of moist vapor, both the strong and the weak solutions of the shock polar are realized. This means that in a moist vapor both regular and Mach reflection of shocks are possible. In other words, all the characteristics of oblique shocks in a moist vapor remain qualitatively the same as for a gas, but the quantitative relations prove to be different.

Thus, the method of calculation of oblique shocks is as follows.

In the case when the given quantities are the static pressure p_1, x_1, number M_1, and angle β_1, we first determine from (9) the value of $(M_1\sin\beta_1)_b$ corresponding to the state of dry saturated vapor behind the shock. If $M_1\sin\beta_1 > (M_1\sin\beta_1)_b$, the shock is calculated according to the variant when the shock process ends in the superheated vapor region. If $M_1\sin\beta_1 < (M_1\sin\beta_1)_b$, the calculation is made according to the variant when the flow of vapor behind the shock is moist.

In the case when the given quantities are static pressure p_1, x_1, number M_1, and flow deflection angle δ, it is necessary to determine the value of $(M_1\sin\beta_1)_b$ from (9), and then the quantity δ_b from (6').

If $\delta > \delta_b$, the calculation is based on the variant when the shock process ends in the superheated vapor region. If $\delta < \delta_b$, the calculation is based on the variant when the state behind the shock corresponds to moist vapor.

In conclusion we note that the equations for calculating normal shocks in moist vapor may easily be obtained as the special case with $\beta_1 = \pi/2$.

NOTATION

p—static pressure; c—stream velocity; T—temperature; R—gas constant; x_1 and x_2—dryness levels before and after shock; k—isentropic exponent; β_1 and B_2—angles between stream direction and shock front; δ—flow deflection angle at shock; r—latent heat of vaporization; c_p—isobaric heat capacity. Subscripts: 1 refers to the state of the flow ahead of the shock, and 2 to the flow behind it.

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